Magnetization of a warm plasma by the nonstationary ponderomotive force of an electromagnetic wave

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It is shown that magnetic fields can be generated in a warm plasma by the nonstationary ponderomotive force of a large-amplitude electromagnetic wave. In the present Brief Report, we derive simple and explicit results that can be useful for understanding the origin of the magnetic fields that are produced in intense laser-plasma interaction experiments.

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Several mechanisms have previously been proposed for the generation of magnetic fields in plasmas, e.g., the nonparallel density and temperature gradients [1], the electron-temperature anisotropy (known as the Weibel instability [2]), the counterstreaming electron beams [3], and the ponderomotive forces of laser beams [4–8]. Spontaneously generated magnetic fields are of great importance in laser produced plasmas [9–12], in our Universe [13,14], and in many cosmic environments [3], as well as in galactic and intergalactic spaces [15–18].

In laser produced plasma experiments [10] and in cosmic plasmas [17] the electrons are heated by lasers and electron beams, respectively. Consequently, there is an electron-temperature anisotropy. Our objective here is to consider the nonlinear interaction between a large-amplitude electromagnetic wave and a hot plasma with electron-temperature anisotropy. We then find that the nonstationary ponderomotive force of the electromagnetic wave creates slowly varying electric fields and vector potentials, which generate magnetic fields. We stress that the present mechanism of the magnetic field generation works only if the plasma has the electron-temperature anisotropy and an electromagnetic field. Thus, in our description one has to calculate the refractive index of the medium by using a kinetic theory [19,20]. It could be pointed out that a previous investigation [21] of the magnetic field generation in the presence of electromagnetic waves has dealt with a fluid description in a *nonuniform* plasma with isotropic electron pressure and reports that the latter is insignificant and that a new solenoidal contribution in a cold plasma emerges due to the time dependence of the product of the wave electric field and the divergence of the product of the wave electric field and the equilibrium electron number density. This result is different from those in Refs. [22–26] which deduced the magnetic fields in a uniform cold plasma on account of the complex nonstationary nonlinear light forces when the wave electric fields are nonplanar.

Let us consider the propagation of an electromagnetic wave, with the electric field $\mathbf{E}(\mathbf{r},t)=(1/2)\hat{\mathbf{x}}E_0(x,t)\exp(-i\omega t+ikz)+$complex conjugate, in an unmagnetized nonrelativistic plasma with an electron-temperature anisotropy $T_\perp/T_\parallel \neq 1$, where $T_\perp$ and $T_\parallel$ are the electron temperature perpendicular and parallel to $\hat{\mathbf{z}}$ and where $\hat{\mathbf{z}}(\hat{\mathbf{x}})$ is the unit vector along the $z(x)$ axis in a Cartesian coordinate system. The ions are immobile. Here $\hat{\mathbf{x}}E_0(x,t)$ is the envelope of the electromagnetic field at the position x and time t. The frequency ω and the wave vector $\mathbf{k}=k\hat{\mathbf{z}}$ are related by a refractive index formula in an anisotropic warm plasma [19,20],

$$\frac{k^2c^2}{\omega^2} = N = 1 - \frac{\omega_{pe}^2}{\omega^2}\left[1 + \frac{T_\perp}{T_\parallel}W(\xi)\right], \tag{1}$$

where c is the speed of light in vacuum, N is the refractive index of the plasma, $\omega_{pe}=(4\pi n_0 e^2/m_e)^{1/2}$ is the electron plasma frequency, n_0 is the electron number density, e is the magnitude of the electron charge, m_e is the electron mass, $W(\xi)=-1-\xi Z(\xi)$, $\xi=\omega/\sqrt{2}kV_{T_\parallel}$, $V_{T_\parallel}=(T_\parallel/m_e)^{1/2}$, and $Z(\xi)$ is the plasma dispersion function [27]. We note that Eq. (1) is simply obtained from the Maxwell equations and the linearized Vlasov equation, with an equilibrium anisotropic electron distribution function [19,20].

The electromagnetic wave exerts a ponderomotive force $\mathbf{F}_p=\mathbf{F}_{ps}+\mathbf{F}_{pt}$ on the plasma electrons, where the stationary and nonstationary ponderomotive forces [4] are

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$$\mathbf{F}_{ps} = \frac{(N-1)}{16\pi}\nabla|E_0|^2 \tag{2}$$

and

$$\mathbf{F}_{pt} = \frac{1}{16\pi}\frac{\mathbf{k}}{\omega^2}\frac{\partial[\omega^2(N-1)]}{\partial\omega}\frac{\partial|E_0|^2}{\partial t}, \tag{3}$$

respectively.

The ponderomotive force pushes the electrons locally and creates the slowly varying electric field,

$$\mathbf{E}_s = -\nabla\phi - \frac{1}{c}\frac{\partial\mathbf{A}}{\partial t} = \frac{1}{n_0 e}\mathbf{F}_p, \tag{4}$$

where the scalar and vector potentials are

$$\phi = -\frac{(N-1)}{16\pi n_0 e}|E_0|^2 \tag{5}$$

and

$$\mathbf{A} = -\frac{c}{16\pi n_0 e}\frac{\mathbf{k}}{\omega^2}\frac{\partial[\omega^2(N-1)]}{\partial\omega}|E_0|^2, \tag{6}$$

respectively.

The induced slowly varying magnetic field $\mathbf{B}_s$ is then $\mathbf{B}_s = \nabla\times\mathbf{A}$. Noting that

$$\frac{\partial[\omega^2(N-1)]}{\partial\omega} = \frac{T_\perp}{T_\parallel}\frac{\omega_{pe}^2}{\sqrt{2}kV_{T_\parallel}}\frac{\partial}{\partial\xi}[\xi Z(\xi)], \tag{7}$$

where N has been replaced by Eq. (1), we can express the magnitude of the magnetic field as

$$|\mathbf{B}_s| = \frac{ecT_\perp|E_0|^2}{4\sqrt{2}m_e L T_\parallel V_{T_\parallel}\omega^2}\frac{\partial}{\partial\xi}[\xi Z(\xi)], \tag{8}$$

where L is the scale length of the envelope $|E_0|^2$. Equation (8) is another expression for the magnetic field in the presence of a large-amplitude electromagnetic wave in a warm uniform plasma with an electron-temperature anisotropy (viz., the temperatures across and along the electromagnetic wave propagation direction are different). It should be stressed that the present mechanism for the magnetic field generation has not been considered in Ref. [21]. The latter reports a magnetic field proportional to the time derivative of $(\mathbf{E}/n_0)\nabla\cdot(n_0\mathbf{E}^*)$, which vanishes for transverse electromagnetic waves ($\nabla\cdot\mathbf{E}^*=0$) in plasmas with constant n_0.

To simplify Eq. (8), we next consider the limit $\xi \gg 1$. Equation (8) then reduces to the electron gyrofrequency formula,

$$\Omega_c = \frac{e|\mathbf{B}_s|}{m_e c} = \frac{k^3 V_{T_\perp}^2 V_0^2}{2L\omega^3}, \tag{9}$$

where $V_0 = e|E_0|/m_e\omega$ is the electron quiver velocity in the electromagnetic field. Equation (9) depicts an interesting scaling of Ω_c against the perpendicular electron temperature $T_\perp$, the wave electric field squared $|E_0|^2$, the laser frequency ω, and the wave number k, as well as the length scale of the laser envelope $|E_0|^2$.

We take some typical parameters that are representative of laser-plasma interaction experiments: $n_0 \approx 10^{20}$ cm^{-3}, $T_\perp \approx 10$ keV, the laser intensity $I = 10^{20}$ W/cm^2, and the laser wavelength $\lambda = 0.25$ μm. For these values, we have $\omega = 8\times10^{15}$ s^{-1}, $V_{T_\perp} = 4.19\times10^9$ cm/s, and $V_0 = 6c\times10^{-10}\lambda\sqrt{I} = 0.15c$. Hence, over the laser wave envelope length of 0.25 μm, the magnetic field strength, estimated from Eq. (9), turns out to be of the order of 60 MG.

To summarize, we have shown that magnetic fields in a warm plasma with an electron-temperature anisotropy can be generated by the nonstationary ponderomotive force of a large-amplitude electromagnetic wave. Specifically, the nonstationary ponderomotive force of the electromagnetic wave pushes the electrons locally and creates slowly varying electric fields and vector potentials. The latter, in turn, produce magnetic fields in a warm plasma with an electron-temperature anisotropy. The latter arises due to the differential electron heating (along and across the electromagnetic wave propagation direction) by an intense laser beam. The present mechanism of the magnetic field generation is relevant for intense short laser pulse-plasma interaction experiments [10,28] and for intense x-ray intense laser pulses interacting with solid density plasma targets [29]. Specifically, spontaneously generated magnetic fields in warm plasmas can affect the electromagnetic wave propagation and the electron energy transport in inertial confinement fusion schemes.

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